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Solar-system tests of gravitational theories

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Annual report to NASA

Principal Investigator: Dr. Irwin I. Shapiro

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We are engaged in testing gravitational theory by means of observations of objects in the solar system. The tests include an examination of the principle of equivalence (POE), the Shapiro delay, the advances of planetary perihelia, the possibility of a secular variation in the gravitational constant, and the rate of the de Sitter (geodetic) precession of the Earth-Moon system.

We have continued to include new data in our analysis as they became available, primarily from the ongoing programs of lunar laser ranging and planetary radar ranging. The accumulation of LLR data, in particular, has increased the power of that data set such that it now outweighs the planetary data in our studies of the possible variation of G. We have made significant progress in the areas listed above, and the results of our investigations have been described in a paper to be submitted to *Physical Review*. Briefly, these results are consistent with the predictions of general relativity, and we obtain the following standard deviations for the relevant parameters: 1×10^{-2} for the fractional relativistic contribution to the perihelion advance of Mercury, 1×10^{-3} for Nordtvedt's η parameter, 2×10^{-2} for the metric parameter β, 2×10^{-3} for the metric parameter γ, and $1 \times 10^{-11} yr^{-1}$ for $\dot{G}/G$. These values are consistent with our preliminary results focusing on the contribution of lunar laser ranging (LLR), which were presented at the seventh Marcel Grossmann meeting on general relativity (Chandler et al. 1996). The largest improvement over previous results comes in η: a factor of five better than our previous value (Chandler et al. 1990). This improvement reflects the increasing strength of the LLR data. Our value for β represents our first such result determined simultaneously with the solar quadrupole moment from the dynamical data set.

Our work has also yielded fruit in related areas. The POE test through the behavior of Earth-borne clocks requires a knowledge of the relative coordinates of those clocks and the sources of the reference timing signals (in our case, distant pulsars), since the usual technique of deducing those coordinates from analysis of the timing signals assumes the validity of POE. The necessary independent information on the pulsar positions (from VLBI observations of the pulsars) provides an opportunity to tie together the reference frame of the extragalactic radio sources and that of the planetary ephemerides. Together with our colleague N. Bartel, we have shown how positions determined from different planetary ephemerides can be compared (a long-standing problem in the field of pulsar timing) and how the combination of VLBI and pulse timing information can yield a direct tie between planetary and radio frames (Bartel et al. 1996).

We anticipate further progress as a result of new data from the ongoing LLR and planetary radar ranging programs, including the newly upgraded Arecibo radar facility, and from further timing and VLBI observations of pulsars, as well as spacecraft tracking data from the Mars Pathfinder now on its way to Mars.

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TOWARD A FRAME TIE VIA MILLISECOND PULSAR VLBI

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ABSTRACT

Millisecond pulsars, by virtue of their relatively stable rotations and sharp pulses of nearly point-source radio emission, can be used to tie Earth-orbit-oriented reference frames to Earth-spin-oriented reference frames with unprecedented accuracy through a combination of timing and VLBI measurements. From 18 cm Mark IIIA VLBI observations on 1987 February 27, we determined the coordinates of PSR B1937+21 relative to those of the extragalactic source 1923+210, whose coordinates in a quasi-inertial extragalactic reference frame have a standard error of ~0.1 mas. These coordinates of PSR B1937+21 at our epoch of observation are: $\alpha(J2000)=19^h39^m38\overset{s}{.}5613\pm0\overset{s}{.}0004$, $\delta(J2000)=21°34'59\overset{''}{.}130\pm0\overset{''}{.}003$. They agree within the root-sum-square standard errors with those determined via VLBI by Petit (1994) and Dewey *et al.* (1996) and with those yielded by the application of the rotation of an indirect frame tie of Folkner *et al.* (1994) to the pulse-timing positions obtained by Kaspi *et al.* (1994) with JPL's DE200 ephemeris. To allow comparison of these positions with pulse-timing positions obtained with respect to other ephemerides, we compute rotation matrices for transforming among the various ephemerides. Pulsar coordinates determined in the frame of one specific planetary ephemeris can now be converted to the frame of any of several other planetary ephemerides or to the IERS frame (via the rotation matrix of Folkner *et al.*) with the conversion procedure given in this paper. Using our own PEP740 ephemeris we obtained a pulse-timing position for PSR B1937+21, and, applying the appropriate rotations, we find agreement within 0.3 mas with that obtained by Kaspi *et al.* with DE200, i.e., within the root-sum-square standard error in each coordinate. We also discuss how a direct frame time tie can be established between these ephemerides and the extragalactic reference frame.

1. INTRODUCTION

The celestial coordinates of pulsars determined by VLBI are related to the Earth's spin vector, and, through other VLBI observations of extragalactic radio sources, to the quasi-inertial extragalactic reference frame. In contrast, the celestial coordinates of pulsars determined via pulse-timing measurements are related to the Earth's orbit. The two sets of positions allow a direct tie to be made between the Earth-orbit-oriented reference frame and the extragalactic reference frame, as first noted by Shapiro (Shapiro & Knight 1970). Such a tie determines, *inter alia,* the obliquity of the ecliptic with corresponding accuracy, and repeated ties allow the rate of change to be monitored accurately (Shapiro 1970). These ties should also be useful in the navigation of interplanetry spacecraft, through tracking with the aid of VLBI.

An indirect tie, based on lunar laser ranging measurements and VLBI observations of extragalactic sources, has been made recently by Folkner *et al.* (1994) with a standard error of ~3 milliarcseconds (mas) about any axis. In contrast, the best direct tie was that obtained by Newhall *et al.* (1986) from VLBI observations of extragalactic sources together with spacecraft either in orbit about Venus or landed on Mars. This tie was uncertain at the level of ~10 mas for one axis and more for the other axes.

Prior VLBI observations of two pulsars, PSR 0329+54 and PSR 1133+16, revealed a significant difference of $0\overset{''}{.}45$ between the pulse-timing and VLBI positions for the former pulsar, but not for the latter (Bartel *et al.* 1985). Similarly, large significant differences were found for many other pulsars between the pulse-timing positions and those determined with the VLA (Fomalont *et al.* 1984). In contrast, no differences were found between the pulse-timing and the VLA

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positions for two precisely timed pulsars, PSR 1913+16 and PSR B1937+21, in excess of the standard error of the VLA positions of 0.″2 and 0.″05, respectively (Backer *et al.* 1985). The latter pulsar has a period of only 1.5 ms and the smallest timing noise of any pulsar yet measured, and its pulse-timing position has consequently been determined with a standard error less than 0.1 mas (Kaspi *et al.* 1994). Therefore, for this pulsar it is particularly useful to compare its pulse-timing position with its VLBI position. Here we report on such a comparison.

2. OBSERVATIONS

We observed the millisecond pulsar PSR B1937+21 and the extragalactic radio source 1923+210 (OV239.7), about 4° away on the sky, in left circular polarization at 18 cm (1.667 GHz) on 1987 February 27. We used the following radio observatories (with telescope diameters, locations, and symbols in parentheses): the National Radio Astronomy Observatory (43 m; Green Bank, WV; G); the Very Large Array (27×25 m; near Socorro, NM; Y); and the Owens Valley Radio Observatory (40 m; Big Pine, CA: O).

At each site, a hydrogen-maser frequency standard was used to govern the local-oscillator chain and the "time tagging" of the recorded data. We used the Mark IIIA VLBI system in mode B (Rogers *et al.* 1983) to obtain a bandwidth of 28 MHz for the GO interferometer, but, because of an error at Y, only 12 MHz for the remaining two interferometers. The observations of the pulsar and the companion source were alternated in a repeated 7 min cycle consisting of a 3 min observation of PSR B1937+21, a 1 min interval for antenna slewing, a 2 min observation of 1923+210, and again a 1 min interval for antenna slewing. In addition, an extra 5 min was allowed for tape changing after each 35 min of cycling.

3. DATA ANALYSIS

All the data were correlated with the Mark IIIA processor at Haystack Observatory yielding both delays and delay rates. To increase the signal-to-noise ratio of the pulsar data, the processor was gated to compute visibility as a function of pulsar phase. We adjusted the width and the phase of the gating window to include only the main pulse of PSR B1937+21. For almost all "scans," gating was essential for detecting the pulsar.

The celestial coordinates of the pulsar were derived by using the time-dependent fringe phase $[0 \leqslant \phi(t) \leqslant 2\pi]$ obtained from each of the three interferometers. For each, we may write

$$\frac{\phi(t)}{2\pi\nu} + \frac{n(t)}{\nu} = \tau(t) = \tau^{g}(t) + \tau^{\rm struc}(t) + \tau^{\rm ion}(t) + \tau^{\rm trop}(t) + \tau^{\rm inst}(t) + \tau^{\rm noise}(t), \tag{1}$$

where ν is the radio frequency to which ϕ is referred, n is the integer number of cycles of phase needed to resolve the phase ambiguity, and $\tau(t)$ is the phase delay. On the right side, τ^{g} is the contribution to $\tau(t)$ from the geometry of the interferometer and the reference point in the source; $\tau^{\rm struc}$ is that from the source structure about this point; and $\tau^{\rm ion}$, $\tau^{\rm trop}$, $\tau^{\rm inst}$, and $\tau^{\rm noise}$ are the contributions, respectively, from the ionosphere, the troposphere, the instrumentation, and the thermal noise. (See, for example, Bartel *et al.* 1986, and references therein for a description of the individual terms.)

To greatly reduce systematic errors in our position estimate due to the uncertainties in τ^{g}, $\tau^{\rm ion}$, $\tau^{\rm trop}$, and $\tau^{\rm inst}$, we formed a differenced phase-delay observable by subtracting from the phase-delay value for each pulsar scan the corresponding value for the subsequent scan of the "reference" source. The standard deviations of the undifferenced and differenced data were set to (separate) constant values for each baseline, so that the corresponding postfit weighted root-mean-square residuals all had approximately the same values. We thus analyzed the undifferenced phase delays for 1923+210, together with the differenced phase delays; for this purpose, we used York University's astrometry software package ASPY, to produce weighted-least-squares estimates of the position (J2000) of PSR B1937+21. [ASPY is a combination of (a) Goddard's CALC package, version 7.6 (Ryan *et al.* 1993), which calculates the theoretical values of τ^{g}; (b) York's interactive phase-connection and plotting package (Ebbers & Bartel 1995), which facilitates the resolution of phase-delay ambiguities; and (c) CfA's VLBI3 program (Robertson 1975), which contains the least-squares-fitting routines.]

We adopted the antenna coordinates, Earth-orientation parameters at the epoch of our observations, and reference-source position from the self-consistent solution produced by the IERS (1993). Although in principle group delays can be combined with phase delays to compute the ionospheric delays (up to an unknown constant for each baseline), PSR B1937+21 was, unfortunately, too weak, and consequently its group delays too uncertain, for this determination to be useful. We therefore employed a simple ionosphere model parameterized by zenith delays that we estimated from Faraday-rotation data (Klobuchar & Royden 1990) for the time of our observations and for locations relatively near the antenna sites (see also Klobuchar & Johanson 1977 and Gwinn 1984). In our model, we adopted values for the total electron content at zenith of 1.5×10^{17} e/m^2 for G and 2×10^{17} e/m^2 for O and Y for the day ionosphere and 0.3×10^{17} e/m^2 for each station for the night ionosphere. In the error analysis below, we allow each of these values to have a 100% standard error. This assumed relative error was consistent with the 1.9 ns root-mean-square variations of the differences between the group and phase delays of 1923+210. For the dry troposphere, we adopted zenith delays from typical values for the surface pressures at the antenna sites. The additional delays due to the wet troposphere were assumed to be typically 0.4 ns for G, 0.3 ns for O, and 0.2 ns for Y. The assumed total tropospheric delays and standard errors were 7.4±0.3 ns for G, 7.0±0.3 ns for O, and 6.2±0.3 ns for Y (Davis 1990). To account for the relative behavior of the station clocks and for certain unmodeled effects, such as deviations from our simple models of the troposphere and ionosphere, we estimated, along with the pulsar position, the coefficients of (*ad hoc*) seventh-order polynomials in time for each site excluding an arbitrary reference site. In addi-

TABLE 1. J2000 position values for PSR B1937+21.[a]

Method	Result α (h m s)		Result δ (° ′ ″)		Result − VLBImean Δα cos δ (mas)	Result − VLBImean Δδ (mas)	Epoch	Reference
VLBI-B	19 39 38.5613	±.0004	21 34 59.130	±.003	...	...	1987.16	this paper
	.5613		.131		1.4	-0.6	1990.0	
VLBI-D	.5611	±.0003	.118	±.016	...	...	1990.2	Dewey *et al.* 1996
	.5611		.118		−1.4	−13.6	1990.0	
VLBI-P	.56123	±.00029	.1323	±.0040	...	...	1992.46	Petit 1994
	.56125		.1334		0.7	1.8	1990.0	
VLBImean[b]	.56120	±.00018	.1316	±.0024	...	...	1990.00	weighted VLBI mean
	.56120		.1316		0.0	0.0	1990.0	
VLA-83	.5613	±.003	.155	±.050	...	...	1983.67	Backer *et al.* 1985
	.5613		.152		1.4	20.4	1990.0	
VLA-90	.560	±.004	.12	±.03	...	...	1990	Fairhead and Backer 1990
	.560		.12		−16.7	−11.6	1990.0	
DE200[c]	.560210	±.000002	.14166	±.00006	...	...	1988.93	Kaspi *et al.* 1994
	.560201		.14116		−13.9	9.6	1990.0	
PEP740[d]	.55596	±.00002	.1416	±.0003	...	...	1986.5	this paper
	.55593		.1400		−73.5	8.4	1990.0	
PEP740R[e]	.51721	±.00002	.3174	±.0003	...	...	1986.5	this paper
	.51718		.3158		-614.0	184.2	1990.0	
PEP740rot.[f]	.56091	±.0002	.1362	±.003	...	...	1986.5	this paper
	.56088		.1346		−4.5	3.0	1990.0	
DE200rot.[f]	.56087	±.0002	.1348	±.003	...	...	1988.93	this paper
	.56086		.1343		−4.7	2.7	1990.0	

[a] To facilitate comparison, each result is given for two epochs, one within the corresponding observing period and another for the approximate mean epoch of the VLBI observations. The position values were propagated to the latter epoch by using the proper-motion values of Kaspi *et al.* 1994: $\mu_\alpha = -0.00932 \pm 0.0006$ ms/yr, $\mu_\delta = -0.464 \pm 0.009$ mas/yr. For the meaning of the standard errors for the VLBI-B entry, see text. All other uncertainties were taken from the original publications.

[b] The weighted mean of the above three VLBI position determinations.

[c] Pulse-timing position determined in JPL's DE200 planetary ephemeris reference frame.

[d] Pulse-timing position determined in CfA's PEP740 planetary ephemeris reference frame. (Data kindly provided by J. Taylor and analyzed by one of us, J.F.C.) The epoch 1986.5 was chosen to make the position and proper motion uncorrelated. For propagating to the mean VLBI epoch, had we used the proper-motion values estimated in our analysis ($\mu_\alpha = -0.01026$ ms/yr, $\mu_\delta = -0.344$ mas/yr), we would have increased the resulting declination by 0.4 mas while changing the right ascension negligibly.

[e] The above PEP740 position transformed to the PEP740R planetary ephemeris reference frame (see text).

[f] Pulse-timing position determinations rotated to the IERS reference frame using the frame ties from Folkner *et al.* (1994) and this paper. See Table 2. The frame tie from Folkner *et al.* applies equally to these determinations and dominates their uncertainties. We therefore show an extra digit for each coordinate to enable comparison.

tion, for each of two of our three sites, we allowed for a constant offset of the phase delays of one source to account for remaining effects of the troposphere and the ionosphere which could not be reduced through modeling and phase-delay differencing. Estimation of these constants reduced the systematic behavior of the postfit residuals in comparison to the behavior of those obtained in an earlier analysis (Bartel *et al.* 1990), in which these constants were not included. We determined the integers $n(t)$ in an iterative fashion, starting from initial values obtained from a fit to the phase-delay-rate data.

Our solution for the position of PSR B1937+21 is given in Table 1. The corresponding residuals of the undifferenced and differenced phase delays are plotted in Fig. 1.

4. ERROR ANALYSIS

The standard errors determined in a weighted-least-squares estimation procedure (0.31 ms in α, 2.8 mas in δ) account for statistical errors in the observations, but perforce not for errors in those parameters of the model that are held fixed. We estimated crudely these latter contributions to the standard error of our solution for each coordinates of PSR B1937+21 by calculating separately the changes introduced in our solution by changes equal to the standard error of each of the important fixed parameters in our astrometric and geodetic models, and then adding the results in quadrature to the statistical error from the original solution. We varied the coordinates of the VLA "phase center" (which have standard errors an order of magnitude larger than those of the coordinates of the other two antennas), obtaining contributions of 0.16 ms and 0.33 mas in the two coordinates, and the ionospheric and tropospheric zenith delays for each antenna, obtaining 0.12 ms and 0.22 mas due to the ionosphere and 0.15 ms and 1.3 mas due to the troposphere. The standard errors of the coordinates of 1923+210 in the IERS reference frame are ~0.1 mas (IERS 1993), and hence negligible. The errors

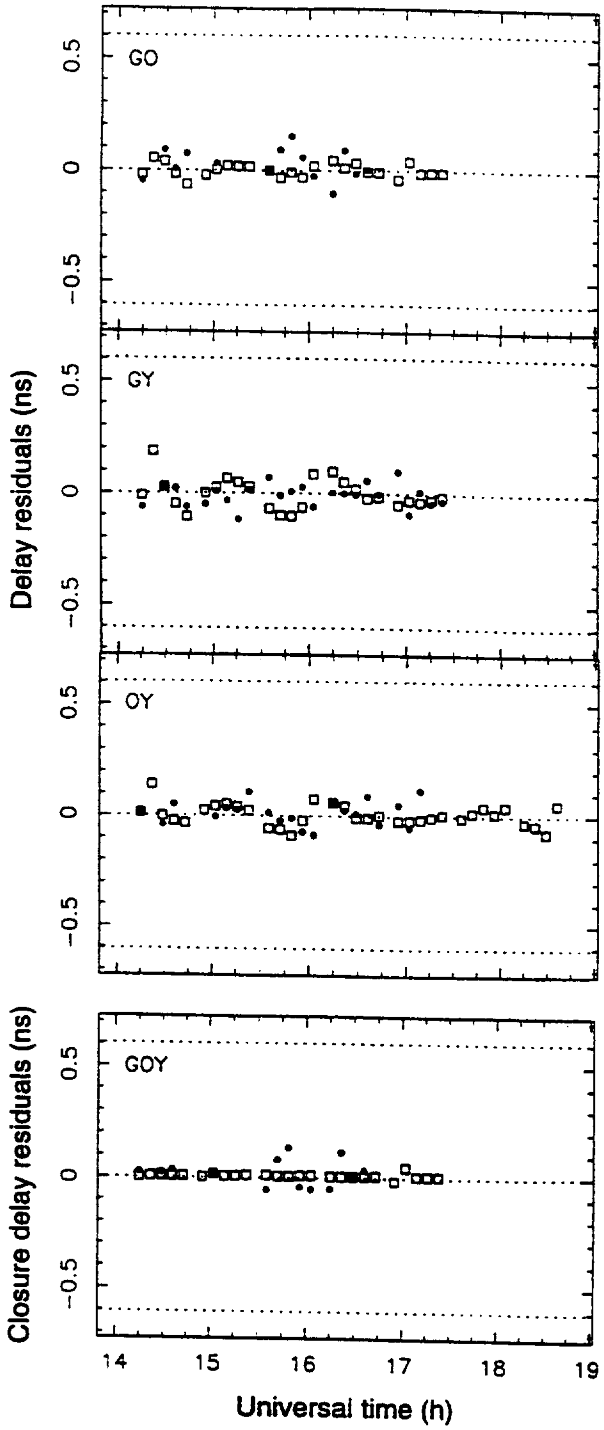

FIG. 1. Upper three panels: Phase-delay residuals for the extragalactic reference source 1923+210 (OV239.7, open squares) and the phase-delay-difference residuals for 1923+210 and PSR B1937+21 (filled circles). Bottom panel: Corresponding closure delay residuals for 1923+210 (open squares) and for the differences for the two sources (filled circles). In contrast to the reference source, the pulsar could not be detected in each scan thus causing the discrepancy in the numbers of the two kinds of residuals. The dashed lines are separated by the delay corresponding to one cycle of phase.

of the other parameters held fixed, namely, UT1, polar motion, and the coordinates of the antennas G and O, were so small (IERS 1993) as to be negligible in their effect on our estimate of the position of PSR B1937+21. To allow for the effect of $\tau^{\rm struc}$, we made a map of 1923+210 from its amplitudes and phases using the AIPS package, found that the source was essentially unresolved, and estimated that the corresponding standard errors for the estimation of the position of the pulsar are much less than 2 mas, and therefore also negligible. Finally, any residual contributions due to $\tau^{\rm inst}$, after it is largely modeled by our polynomials, are assumed to be negligible, since the use of polynomials of sixth degree rather than seventh results in insignificant changes to our position estimate. The estimated standard errors are given in Table 1 for our pulsar coordinates at our observing epoch as the root-sum-squares (rss) of the above error contributions.

In support of our use of the method of weighted least-squares, without allowing for correlations in the data, we note that little, if any, correlation is evident in Fig. 1 between the residuals of the differenced and undifferenced data. However, there appears to be a modest correlation between successive points in the undifferenced delay residuals shown in the upper panels of the figure. Moreover, some correlation among simultaneous delays on the three baselines is implied by the relatively small scatter in the closure-delay residuals in the bottom panel. Nevertheless, since the signature of an error in our pulsar position is a diurnal sinusoid in the *differenced* delay residuals, which exhibit less, if any, correlation, the more complicated analysis required to explicitly account for correlated noise in the data is not justified. We estimate that the effect of such correlations on our statistical standard errors would be an increase of less than 20%, and that our position estimate would change by only a small fraction of its standard error.

5. COMPARISON WITH OTHER POSITION DETERMINATIONS

In Table 1 we list other VLBI and pulse-timing determinations of the position of PSR B1937+21. To facilitate comparison, we propagated each position determination to epoch 1990.0 using the proper-motion value of Kaspi *et al.* (1994), and computed the weighted mean of each coordinate of the VLBI positions. In Table 1 we also present the offset of each position determination from this mean VLBI position. We plot these offsets in Fig. 2(a), and for clarity we replot at a larger scale the tightly clustered points, together with the mean VLBI position, in Fig. 2(b).

Our VLBI position determination agrees in both coordinates with those of both Petit (1994) and Dewey *et al.* (1996) to within the respective combined standard errors. Further, as expected, our VLBI position agrees with the two VLA positions within their much larger standard errors (Backer *et al.* 1985; Fairhead & Backer 1990).

To compare properly the VLBI position estimates and those from pulse timing, we must put all the coordinates in the same reference frame. Indeed, if we compare the coordinates without these corrections, we find seemingly significant, but misleading differences. For JPL's DE200 (Standish 1986), the offset ($\Delta\alpha \cos\delta$, $\Delta\delta$) from the mean VLBI position is (−14, 10 mas), while for CfA's PEP740 it is (−74, 8 mas). The offset is even larger for PEP740R. These differences are not surprising. Each position determined from pulse-timing analysis is strictly tied to the orientation of

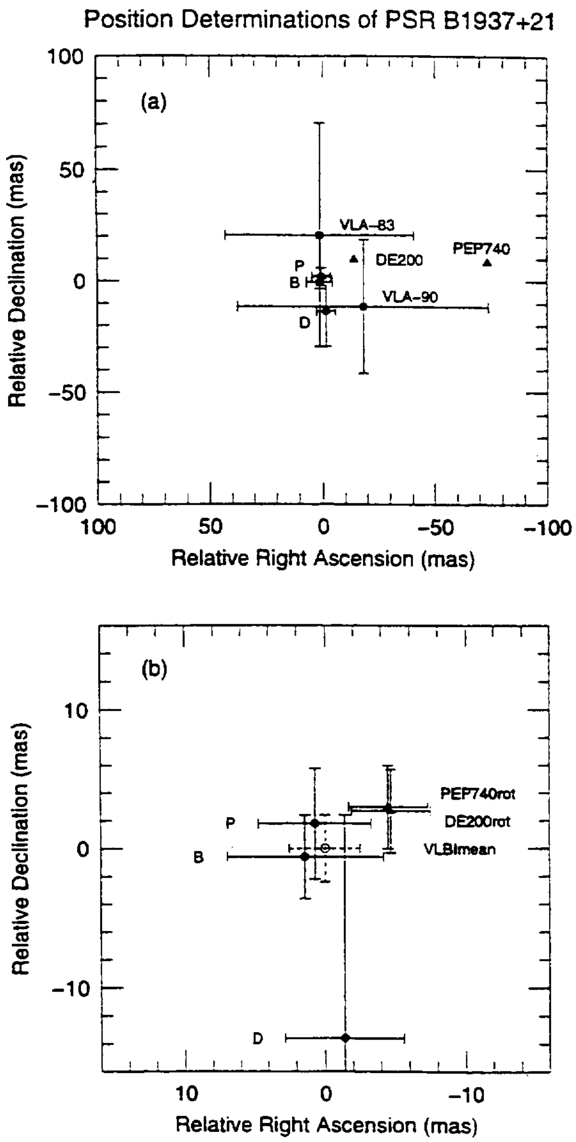

FIG. 2. (a) Positions of PSR B1937+21 determined with VLBI (B: this paper; D: Dewey *et al.* 1996; P: Petit 1994), with the VLA (VLA-83: Backer *et al.* 1985; VLA-90: Fairhead & Backer 1990), and with pulse timing, the last analyzed with two planetary ephemerides (JPL's DE200 and CfA's PEP740). All positions are plotted relative to the weighted mean of the VLBI positions (VLBImean). See Table 1 for the numerical values. Note that the difference in right ascension between DE200 and PEP740 is due almost entirely to a difference in the (arbitrary) origin of right ascension. The tabulated uncertainties are so small that the error bars would be invisible on this scale. (b) The same VLBI results replotted with an expanded scale. Also shown are their weighted mean (dashed) and the DE200 and PEP740 positions rotated to the IERS frame (DE200rot and PEP740rot).

Earth's orbit with respect to the coordinate axes of the planetary ephemeris used. We note that the scientific objective for our development of planetary ephemerides has been the study of solar-system dynamics rather than a tie to an extragalactic reference frame. For such work, the orientation of the solar system with respect to the extragalactic frame is irrelevant. Indeed, that orientation depends on the set of data used in producing the ephemeris (and on the coordinate system used in the theoretical framework). These three ephemerides were produced with quite different schemes for establishing the orientation. PEP740 was tied primarily to Earth's spin axis through lunar and planetary ranging data and (for historical reasons) indirectly to a Galactic reference frame through two centuries of optical astrometric observations of Saturn, Uranus, and Neptune via the perturbations of those planets on Earth and Mars. PEP740R is a vector rotation of PEP740 to agree at 1982.9 with our nearly 30-year-old PEP311 ephemeris, whose orientation with respect to Earth's axis and the Galactic frame was determined primarily by optical observations of the Sun, Moon, and planets. DE200 was based on a different set of optical data, including none obtained earlier than 1911 (Standish 1986, 1990), and was rotated so that its origin of right ascension is on its own dynamical equinox of J2000 (Standish 1982).

We produced PEP740R in 1984, not because of any intrinsic advantages of its coordinate frame, but as a convenience for pulsar observers, who were at that time still using PEP311 and wished to minimize the offset in calculated pulse-timing positions produced by adopting the new (PEP740) ephemeris. Indeed, we could omit all discussion of PEP740R and PEP311, except that we wish to provide it for possible use by the many authors who have used one or the other in deriving pulsar positions. We therefore demonstrate here that pulse-timing positions based on these ephemerides can be converted readily to more modern reference frames. The old optical data that determined the orientation of PEP311 have systematic errors up to an arcsecond, and that ephemeris therefore differs correspondingly in orientation from the more modern ones. Thus, we focus on the more recent PEP740 and DE200 ephemerides, but consider the others (see below) for the reasons cited above.

To make more useful comparisons among pulse-timing positions, we first compare the ephemerides directly by fitting the heliocentric coordinates of the Earth–Moon barycenter of one ephemeris to those of another in a least-squares sense. For each pair, we solve for the parameters that represent a vector rotation and rate that transform between the ephemerides. We include ephemerides that have been cited in pulsar position determinations or are otherwise relevant to a global frame tie, namely, DE96, DE118, and DE200 (all from JPL) and PEP311, PEP740, and PEP740R (all from CfA or, earlier, from MIT and MIT Lincoln Laboratory). These ephemerides, except for DE200, have their x, y, and z axes approximately aligned with the directions specified by B1950.0 coordinates (0^h, 0°), (6^h, 0°), and (0^h, 90°), respectively. [By using the term "B1950.0," as opposed to "B1950," we imply the new IAU system, including, in effect, the equinox of Fricke (1982).] The axes of DE200 are approximately aligned with J2000.0 coordinates (0^h, 0°), etc. To focus upon the small rotations required to fit any one ephemeris to the others after they are all rotated by the IAU 1980 precession (Lieske 1979) to a common epoch, we apply the rotation from B1950.0 to J2000.0 axes before performing the comparisons, thereby obtaining subarcsecond differences in all cases. The postfit root-mean-square difference between the coordinates in any two of these ephemerides is on the order of 10^{-10} AU; the discrepancies could contribute at most about 10^{-10} rad (0.02 mas) to differences among pulse-timing positions. We found equivalent results with two methods: (a) fitting an overall rotation and rate for a twelve-year span, and (b) fitting a rotation alone for each year and then

TABLE 2. Frame conversions via Ephemeris comparisons.[a]

	Rotation (10^{-10} rad) at 1990.0			Rate (10^{-14} rad per day)		
	A_1	A_2	A_3	$\dot{A}_1$	$\dot{A}_2$	$\dot{A}_3$
PEP740 to PEP740R	13607	-9064	33698	0	0	0
PEP740 to PEP311	13606	-10143	36183	-4	-4087	9411
PEP740 to DE96	3948	1681	13587	8	2729	-6304
PEP740 to DE118	-195	297	22495	2	26	-65
PEP740 to DE200	-70	296	-3239	2	26	-65
PEP740 to IERS[b]	-170	-280	-3530	...	...	...

Notes to Table 2

[a]The standard errors in all the rotations derived from direct ephemeris comparison are at the 10^{-10} rad level, and those for the rates are at the 2×10^{-14} rad/day level. These standard errors are approximately the root-mean-square residuals of the ephemeris comparisons. The results are presented as vectors in the J2000 frame.

[b]Obtained from the preceding line for PEP740-to-DE200 by adding the DE200-to-IERS rotation vector from Folkner et al. (1994). The time dependence of the latter was not determined, but was assumed to be dominated by the error in the mean motion of the Earth in the DE200 ephemeris. By direct comparison with the Earth's motion in a more recent ephemeris, Folkner et al. infer a mean-motion error of ~0.5 mas/yr, a value that we can marginally ignore for our present purposes, since the mean observing epochs of the pulse-timing determinations (1985.0, 1986.5, and 1988.93) fall within ~3 years of the ~1988 epoch of Folkner et al.'s tie. The standard error of the PEP740-to-IERS rotation vector is dominated by that of the DE200-to-IERS rotation.

obtaining a slope from this set of fits. We present the results in Table 2. Since the rotation angles are very small, we can with negligible error approximate each corresponding rotation matrix as

$$M=\begin{pmatrix} 1 & A_3 & -A_2 \\ -A_3 & 1 & A_1 \\ A_2 & -A_1 & 1 \end{pmatrix},$$

where the A_i are the time-dependent elements of the vectors obtained in our fits; these elements are shown in Table 2. Given a vector $\mathbf{r}^I$ based on ephemeris "I" and the rotation matrix $M^{I,J}$, we can predict the corresponding vector $\mathbf{r}^J$ for ephemeris "J" by the matrix multiplication

$$\mathbf{r}^J = M^{I,J}\mathbf{r}^I,$$

where $\mathbf{r}^I$ and $\mathbf{r}^J$ are expressed as column vectors. These transformations can be combined by matrix multiplication or, more simply, by adding or subtracting the corresponding elements A_i. Where relevant, the precession (without any other corrections) should also be applied.

We use these transformations to obtain the coordinates shown on the last two lines of Table 1, i.e., the two pulse-timing positions (obtained by us and by Kaspi *et al.*) reduced to the IERS reference frame, PEP740rot and DE200rot. These two positions had been obtained independently with different ephemerides from analyses of overlapping (though not identical) data sets (see notes to Table 1). The two reduced positions agree to within their rss standard errors. Similarly, we find that our B1950.0 PEP740R position for the pulsar agrees within one standard error with the B1950.0 positions obtained by Davis *et al.* (1985) and Rawley *et al.* (1987) using PEP740R. On the other hand, we find a disagreement (−630, 70) mas between our J2000 PEP740R position (shown in Table 1) and that given by Kaspi *et al.* (1994), which they obtained from their B1950.0 PEP740R position by ignoring the particular orientation of PEP740R and applying Fricke's equinox correction as if that position were on the old IAU system (Kaspi 1995).

The pulse-timing positions PEP740rot and DE200rot agree with each of the VLBI positions and their mean approximately within the corresponding rss standard errors. (The agreement is within the rss standard error in all cases except that of Petit's right ascension, which differs from the timing positions by about 1.1 rss, and that of the VLBI mean right ascension, which differs from the timing positions by about 1.3 rss.) We view this level of agreement as a successful test of Folkern *et al.*'s frame tie and our method of comparing positions.

6. EXAMPLES OF FRAME CONVERSION

We present here some examples of this transformation process. We start with the pulsar coordinates from Rawley *et al.* (1987) given with respect to the PEP740R ephemeris and convert them to the corresponding coordinates in the IERS reference frame. The published position ($19^h37^m28^s.74601$, $21°28'0''.4588$) is first converted to a unit vector and then precessed from B1950.0 to J2000.0 by multiplying by the IAU 1980 precession matrix. To the same level of precision as the rotations in Table 2, that matrix is

$$\begin{pmatrix} 0.9999257080 & -0.0111789381 & -0.0048590038 \\ 0.0111789381 & 0.9999375133 & -0.0000271626 \\ 0.0048590038 & -0.0000271579 & 0.9999881946 \end{pmatrix}.$$

The precessed unit vector corresponds to coordinates ($19^h39^m38^s.51726$, $21°34'59''.3194$). The transformation from PEP740R to IERS is then found by subtracting the first line of Table 2 from the last, giving $(A_1, A_2, A_3) = (-13777, 8784, -37228)\times10^{-10}$ rad. This transformation is time dependent in principle, but the rate is undetermined for the last line of Table 2 (see footnote *b* to table). Substituting these values into the expression for M and applying M to the J2000.0 unit vector gives a final unit vector referred to the IERS frame, corresponding to ($19^h39^m38^s.56097$, $21°34'59''.1380$). To compare the transformed position with those in Table 1, we also apply a correction for proper motion from epoch 1982.9 to epoch 1990.0. If we use the proper motion determined by Rawley *et al.* to change epoch from 1982.9 to roughly the midpoint of their timing data (about 1985.0) and then use the better determined proper motion of Kaspi *et al.* (1994), we find a correction of ($-0^s.00009$, $-0''.0034$), which gives close agreement with both of the positions in Table 1 referred to the IERS frame.

As a second example, we take the position obtained by Kaspi *et al.* with respect to the DE200 ephemeris (shown in Table 1) and convert to the frame of PEP740R. Subtracting the relevant lines of Table 2, we get a vector for 1990.0 of $(13677, -9360, 36937)\times10^{-10}$ rad and a rate of $(-2, -26, 65)\times10^{-14}$ rad/d. At the specified epoch of 1988.93, the vector is $(13677, -9359, 36934)\times10^{-10}$ rad. Applying that rotation and then the transpose of the precession matrix, we get coordinates referred to the frame of PEP740R ($19^h37^m28^s.745917$, $21°28'01''.45559$). As in the first example, after correcting for proper motion we find close agreement with the PEP740R position of Rawley *et al.*

As a final example, we convert the same position to the IERS frame. Again, as in the first example, the rotation rate between the two frames is undetermined, and we ignore it. Differencing the last two lines of Table 2, we get a vector rotation of $(-100, -576, -291)\times10^{-10}$ rad. Applying this rotation yields ($19^h39^m38^s.560871$, $21°34'59''.13479$).

7. PROSPECTS

An improvement in the accuracy of the determination of the position of PSR B1937+21 in the extragalactic reference frame appears feasible. We believe an accuracy of 1 mas in both α and δ could be obtained with VLBI by using one or two extragalactic reference sources located on the sky substantially closer to PSR B1937+21 than the 4° of 1923 +210. Such closer reference sources are now known, but were not known in early 1987.

If the position of another millisecond pulsar suitably located far away on the sky from PSR B1937+21 could be determined with comparable accuracy, then the rotation parameters for the frame tie (including the mean obliquity of the ecliptic) could be determined with milliarcsecond accuracy. Alternatively, a new planetary ephemeris could be placed directly on the extragalactic reference frame by generating the ephemeris with PSR B1937+21's VLBI position determination as a constraint. Spanning a few years, VLBI observations of two pulsars could determine the annual rate of change of the frame tie with comparable accuracy. Although it is currently believed based on theory that the orientation of the extragalactic and ephemeris frames are fixed with respect to each other to submilliarcsecond accuracy, continued VLBI and pulse-timing observations of pulsars could provide the first direct test of this belief.

PSR B1937+21 is also a good candidate to be used as part of a measurement of the annual variation in the gravitational redshift of Earth-borne clocks predicted by general relativity, through combined analysis of VLBI and pulse-timing observations (Shapiro 1986; Chandler 1990). If the position of PSR B1937+21 could be determined with an accuracy of 1 mas, the signature of the gravitational redshift on terrestrial clocks could be measured with a fractional accuracy of about 1 part in 10^3. Such a test of the gravitational redshift would be about an order of magnitude less accurate than the most accurate test to date (Vessot *et al.* 1980), which probed the Earth's potential, but would be by at least an order of magnitude the most accurate test of the redshift due to the solar potential.

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